

# Atmosphere Infrared Absorption Model, Atmospheric Window Formula and Atmospheric Rayleigh Dispersion Formula

Xin-an Zhang

Xi'an Polytechnic University, Xi'an, 710048

**Abstract:** In this paper, electromagnetic wave absorption principle of water vapour molecules in atmosphere has been analyzed. As a result, the molecular electromagnetic wave absorption formula of water vapour has been established. The absorption-transmitted spectrum calculated from this formula is well agreement with atmospheric window. Consequently, the conclusion is deduced that the water vapour in atmosphere make the main contribution to the absorption of electromagnetic wave and the molecular electromagnetic wave absorption formula of water vapour is just the atmospheric window formula. In addition, the Rayleigh dispersion formula in atmosphere has been developed as well. Finally, the law of varying of electromagnetic wave transmittance caused by water vapour content has been discussed.

**Keywords:** electromagnetic wave, molecular absorption principle, atmospheric window formula, atmospheric Rayleigh dispersion formula, water vapour content

## 0. Introduction

The transmitting and decay of infrared in the atmosphere are related to satellite communication, aviation remote sensing, radar, infrared radiation survey, etc<sup>(1-4)</sup>. They play the most important role in the high technology in military. The model of the transmitting and decay of infrared in the atmosphere is the key in the infrared image formation guidance system. They are also related in photoelectricity resistance system, split-ring test to the guiding head, search and track algorithm<sup>(4-7)</sup>. As the military technology has been applying in the use of civil, the transmitting and decay of infrared in the

atmosphere will be more wide use filed.

## 1. The atmosphere absorption principle of electromagnetic wave

From traditional idea, in atmosphere, the molecules which absorb the infrared are mainly:  $H_2O$ ,  $CO_2$ ,  $O_3$ ,  $N_2O$ ,  $CH_4$ ,  $CO$ .

While, the vapour ( $H_2O$ ), carbon dioxide ( $CO_2$ ), ozone ( $O_3$ ) make the main contribution<sup>(1-3)</sup>.

It is assumed that the atoms in the  $H_2O$  molecule are arranged as in figure 1. The electric field of the electromagnetic wave is changed as the sine function which have been shown under one hydrogen atom

If the amplitude of the electric field wave is the positive pole, the hydrogen  $H_1$  will be attracted on the ordinate which is the negative pole of electric field. Then, the positive pole (amplitude) will push the  $H_2$  moving to X (right) direction. While when it turns to negative direction,  $H_2$  will be bring back to the original position by the attraction of the electrons between two hydrogen atoms nuclear.

Due to the fact that the moving of  $H_2$  to the X' direction will be resisted by  $H_1$ ,  $H_2$  will be stopped at the original position. That means, only the positive amplitude of the electric field can make the work to the atom  $H_2$ . The consuming action of the electric field to the

molecule will be similar to an intermittence pulse ( see figure 2). According the vibration knowledge <sup>(8)</sup>, we can get the moving equation of the hydrogen atom  $H_2$ . Let a force with any period acting on a single oscillator, this force can be expressed as  $F_F(t) = F_F(t + kT)$  ( $k = 0, 1, 2, 3 \dots$ ) (1) Under the acting of this force, the vibration equation of oscillator particle will be

$$M_m \frac{d^2 \xi}{dt^2} + R_m \frac{d \xi}{dt} + K_m \xi = F_f(t) \quad (2)$$

Where,  $\xi$  is the displacement of oscillator particle.

Make Fourier launch to Eq. (1) and let

$$F_n = \sqrt{A_n^2 + B_n^2}, \quad \varphi_n = \arctan \frac{B_n}{A_n} \quad (3)$$

$$A_n = F_n \cos \varphi_n, B_n = F_n \sin \varphi_n \quad (4)$$

Considering the complex expression

$$F_F(t) = \sum_{n=3}^{\infty} F_n e^{j(n\omega t - \varphi_n)} \quad (5)$$

$$\text{and} \quad \xi = \sum_{n=3}^{\infty} \xi_n \quad (6)$$

There will be

$$\xi_n = \frac{F_n}{jn\omega Z_n} e^{j(n\omega t - \varphi_n)} \quad (7)$$

Where

$$Z_n = R_n + jX_n = R_m + j(n\omega M_m - \frac{K_m}{n\omega})$$

$$\text{Considering } Z_n = \sqrt{R_m^2 + (n\omega M_m - \frac{K_m}{n\omega})^2},$$

$$\theta_n = \arctan \frac{X_n}{R_m} \text{ and, let the expression of}$$

force with the period shown in figure 2

$$F(t), \quad kT \leq t \leq (k + \frac{1}{2})T$$

$$F_F(t) = \begin{cases} 0, & (k + \frac{1}{2})T \leq t \leq (k + 1)T \\ (k = 0, 1, 2, 3 \dots) \end{cases} \quad (8)$$

we get

$$A_0 = \frac{F(t)}{2}, \quad A_1 = A_2 = A_3 = \dots = 0$$

$$B_1 = \frac{2}{\pi} F(t), \quad B_2 = 0$$

$$B_3 = \frac{2}{3\pi} F(t), \quad B_4 = 0 \dots$$

That is

$$\frac{2}{n\pi} F(t), \quad n \text{ are odd number}$$

$$B_n = \begin{cases} 0, & n \text{ are even number} \end{cases}$$

Finally,

$$\begin{aligned} \xi = & \frac{F(t)}{2K_m} + \frac{2F(t)}{\omega\pi|Z_1|} \cos(\omega t - \theta_1 - \pi) + \\ & \frac{2F(t)}{9\pi\omega|Z_3|} \cos(3\omega t - \theta_3 - \pi) + \dots \\ & \frac{2F(t)}{n^2\pi\omega|Z_n|} \cos(n\omega t - \theta_n - \pi) \end{aligned} \quad (9)$$

$$\text{Where, } \omega = \frac{2\pi}{T}, \quad n \text{ are odd number}$$

This equation shows that the particle will get the static displacement under the acting of pulse force. In addition, there will also be a series odd number resonance frequency. That is, these resonance vibration frequencies will increase in the odd number times of the basic frequency.

## 2. Atmospheric window formula

From the analyzing above, the vibration of the atoms in water vapour molecule will just absorb the electromagnetic wave in one side in the intermittence pulse way. This process lead to that the resonance of molecule vibration and the electromagnetic wave will just occur in the frequencies of the odd times of basic molecule vibration frequency. In other words, the electromagnetic wave will be absorbed in the frequency with the odd number times of the basic frequency of molecules. According to this conclusion, the electromagnetic wave absorption and transmittance will be established.

Let the electromagnetic wave length

corresponding to the basic vibration frequency of hydrogen atom be  $\lambda_0$ . To make the absorption frequency increase in the odd times of basic frequency of molecule, the cosine function is needed. When the cosine function equals to 1, the electromagnetic wave will be totally transmitted. When the cosine function equals to -1, the electromagnetic wave will be totally absorbed. Add an extra item to the cosine function, after been corrected, the atmospheric window will be

$$T = 0.77 \cos\left(\frac{\lambda_0}{\lambda} \pi\right) + 0.23 \quad (10)$$

Where,  $T$  is transmittance,  $\lambda_0 = 20.00 \mu m$  is the electromagnetic wave length corresponding to the basic vibration frequency of hydrogen atom,  $\lambda$  ( $\mu m$ ) are the electromagnetic wave length of all light. If the equation equal to the positive number, it represents the transmission of electromagnetic wave. If the equation equal to the negative number, it represents the absorption of electromagnetic wave.

Figure 3 is the famous atmospheric window which has been quoted by most of literature of basic optics and remote sensing<sup>(1-3,7)</sup>. The calculation results from formula (10) have been shown in table 1 and figure 4. Comparing to the figure 3 with table 1 and figure 4, it is seen that calculation results from formula (10) is well just agreement with the atmospheric window. Consequently, the conclusion is deduced that the water vapour in atmosphere make the main contribution to the absorption of electromagnetic wave and the atmospheric window formula is just the molecular electromagnetic wave absorption formula of water vapour.

It is also concluded from formula (10) that the resonance frequency electromagnetic wave and the molecule are  $1.49 \times 10^{13}$ ,  $4.49 \times 10^{13}$ ,

$7.48 \times 10^{13}$ ,  $10.49 \times 10^{13}$ ,  $13.51 \times 10^{13}$ ,  $16.48 \times 10^{13}$ ,  $19.48 \times 10^{13}$ ,  $22.39 \times 10^{13}$  ... which corresponding to the absorption peak. Their multiply relation is 3, 5, 7, 9, 11, 13, 15... Therefore, the basic vibration frequency of hydrogen atom in the water vapour is  $1.4949632 \times 10^{13}$  Hz.

### 3. Atmospheric Rayleigh dispersion formula

According to the discussion above, the mainly absorption matter in the atmosphere to the electromagnetic wave is water vapour molecules. Therefore, the dispersion matter in the atmosphere to the electromagnetic wave is also mainly water vapour molecules. Due to the size of water vapour molecules is far smaller than infrared and visible light, the Rayleigh dispersion formula can be applied to water vapour molecules. The Rayleigh dispersion formula is

$$I = \frac{e^2 x_0^2 \omega^4}{32 \pi^2 \epsilon_0 c^3 R^2} \sin^2 \alpha \quad (11)$$

Where,  $e$  is the electric charge of atom,  $\omega$  is the vibration frequency of atom,  $x_0$  is the

amplitude of vibration,  $c$  is light speed,  $\epsilon_0$  is the electric capacity,  $R$  is the distance from observer to molecule,  $\alpha$  is the angle of observe.

From Eq.(9), we know the displacement of the hydrogen atom which is also the amplitude of vibration of molecule. Then the Rayleigh dispersion formula of water vapour molecule, which can also be named as atmospheric Rayleigh dispersion formula can be approached

$$I = \frac{e^2 \omega^4 \sin^2 \alpha}{32 \pi^2 \epsilon_0 c^3 R^2} \left[ \frac{F(t)}{2K_m} + \frac{2F(t)}{\omega \pi |Z_1|} \cos(\omega t - \theta_1 - \pi) + \frac{2F_a(t)}{9\pi \omega |Z_3|} \cos(3\omega t - \theta_3 - \pi) + \dots \right]$$

$$\frac{2 F_a(t)}{n^2 \pi \omega |Z_n|} \cos(n \omega t - \theta_n - \pi)]^2 \quad (12)$$

Where,  $\omega = \frac{2\pi}{T}$ ,  $n$  is odd number

#### 4. The relationship between water vapour content and the transmittance of infrared

Because the mainly absorption matter in the atmosphere to the electromagnetic wave is water vapour molecules, the water vapour content will affect the transmittance of light in atmosphere. Tabel 2 show the measured data of transmittance of infrared at different level distance from the sea level.

From Eq.(12), it is seen that the dispersion intensity of light will increase with the increasing of frequency. That means, in the small wavelength, the dispersion intensity of electromagnetic wave will be stronger. Then, the measured transmittance of light which consist some dispersion of light will be obviously strong. That is, in the small wavelength, the measured transmittance will be larger than the real transmittance of water vapour. In table 2, it can be seen that, the measured transmittance decrease obviously in the long wave length with the decrease of water vapour content. Hence, in table 2, the transmittance in the long wave length reflects the real situation of the transmittance and absorption of water vapour to electromagnetic wave. While, in the small wavelength, the dispersion intensity is very high, which add on the real transmittance and enlarge the measured results.

Considering the effecting of water vapour content, in the Eq.(10), the last constant item will a function of water vapour content. Drawing the law from table 2, the Eq.(10) will be changed to

$$T = 0.77 \cos\left(\frac{\lambda_0}{\lambda} \pi\right) + 0.3 w^{-0.03 \lambda} \quad (13)$$

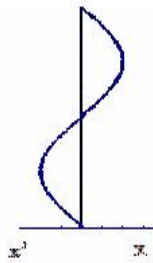
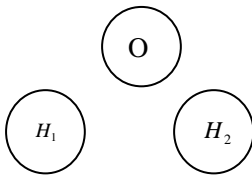
Where,  $w$  is water vapour content (mm)

#### 5. Conclusions and discussions

The author has put forward an electromagnetic wave absorption model of water vapour in the atmosphere. Consequently, the electromagnetic wave absorption formula of water vapour has been established. The spectrum calculated from this formula is well agreement with the atmospheric window, which deduce a conclusion that the main absorption matter in the atmosphere to the electromagnetic wave is water vapour molecules and the atmospheric window formula has been found. In addition, the Rayleigh dispersion in the atmosphere is also mainly caused by the water vapour molecules. Then, the Rayleigh dispersion formula of atmosphere has been obtained. Finally, the law that the water vapour content decreases the transmittance of infrared has been discussed.

#### Reference

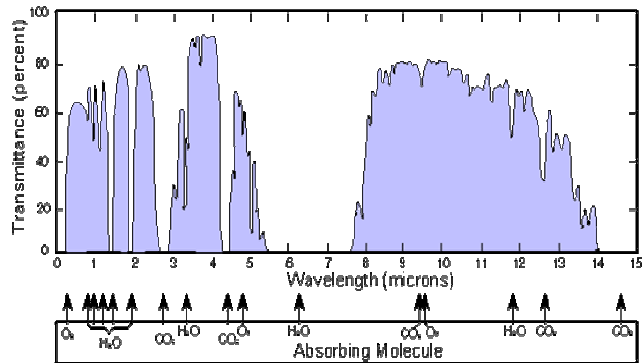
- [1] Richard D.Hudson,JR. Inferared system engineering, Chinese translation, Beijing: Defense industry Press,1975, 69-101
- [2] Li,X.W., The principle and application of remote sensing, Beijing: Science Press,2008, 33-35
- [3] Yi.J.E., The intrucuction of modern remote sensing, Beijing: Science Press, 2008, 33
- [4] George E Caledonia. Infrared Radiation Produced in Ambient/ Spacecraft - Emitted Gas Interactions under LEO Conditions[ Z] .AIAA 00 - 0104 (2000) .
- [5] Sun.Z.L., Manual of infrared and Photoelectricity system(5<sup>th</sup>), Tianjin: Astronautics industry company 3,8358 institute translate and press,2001.
- [6] Zai.,Q.H.,Wang,Q., Numerical Simulation of IR Image of High-speed Aircraft,Laser & Infrared ,2002 ,32 (3) :146 - 148.
- [7] H.A.Gebbie et al., Atmospheric Transmission in the 1 to 14  $\mu$  Region,Proc. Roy.Soc.,A206,87(1951)
- [8] Du.,G.H., Zhu.,Z.M.,Gong, X.F. Basic acoustics. Nanjing: Najing University Press, 2001: 42~47



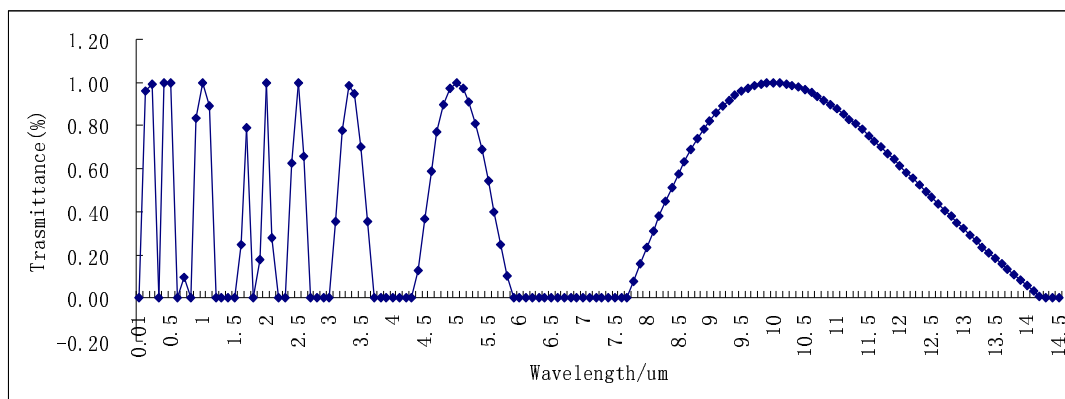
**Figure 1** The interaction between electromagnetic wave and molecule



**Figure 2** Pulse process



**Figure 3** Atmospheric window<sup>(3,5,6,15,16)</sup>



**Figure 4** The calculation results of atmospheric absorption to electromagnetic wave

Note: For the convenience of comparing with figure 3, all of the negative data calculated from Eq. (10) been taken as zero

**Table 1** The theory and measurement data of the absorption wavelength of electromagnetic wave in atmosphere

|              |       |         |         |         |         |         |         |       |
|--------------|-------|---------|---------|---------|---------|---------|---------|-------|
| Theory       | 0-0.1 | 1.2-1.5 | 1.8     | 2.7-3.0 | 3.7-4.3 | 3.7-4.3 | 5.9-7.8 | >14.3 |
| Measurements | 0-0.2 | 1.3-1.5 | 1.7-1.8 | 2.6-2.8 | 4.1-4.3 | 3.8-3.9 | 5.5-7.7 | >14   |

**Table 2** The measured transmittance of infrared at different level distance from the sea level<sup>(1)</sup>.

| Water vapour content<br>Level distance<br>Wavelength | 6750mm<br>90000 feet | 2700mm<br>36000 feet | 1350<br>18000 feet | 675mm<br>9000 feet | 270mm<br>36000 feet |
|--|----------------------|----------------------|--------------------|--------------------|---------------------|
| 1.5 $\mu m$  | 0.91                 | 0.94                 | 0.95               | 0.96               | 0.97                |
| 2.2 $\mu m$  | 0.92                 | 0.94                 | 0.96               | 0.98               | 1.00                |
| 3.9 $\mu m$  | 0.93                 | 0.95                 | 0.96               | 0.98               | 1.00                |
| 4.6 $\mu m$  | 0.25                 | 0.45                 | 0.59               | 0.71               | 0.81                |

Note: the wavelength in the table are corresponding to the peak transmittance of infrared